

The bronze chime bells of the Marquis of Zeng: Babylonian biophysics in Ancient China

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The recent recovery of a well-tuned carillon of sixty-five bronze bells, from a tomb dating to 433 BC, is one of the most startling musical archeological events of all time. The five-octave, chromatic chime-set, with most of its bells tuned to the 'philosophical pitch standard' (middle C = 256 cps), is a sophisticated achievement beyond the reach of Western science and technology for another two thousand years. Some 2800 characters inscribed on the bells and the bell-stand name the tones within both a 'relative pitch' system and an 'absolute pitch' system, the latter in the key of A-flat, related directly to the calendar. Alternate tunings together with the inscriptions suggest that this funerary set may have been conceived as both an acoustical laboratory and a musicological library. Coincidences between abstract Chinese tone ratios and measured frequencies of the largest bells suggest that Zeng acousticians may have been able to make accurate frequency determinations. Patterns in both the tuning and the naming system, never known to historians, now show that in the fifth century BC the tone systems of China and Greece shared extensive structural similarities. This new archeological evidence thus strengthens the hypothesis that an ancient Mesopotamian biophysics linking number, tone, and calendar enjoyed a pan-Eurasian development.

Part 1: Archeological background

Introduction: A Chinese musical treasure

From a tomb discovered accidentally by a construction crew in Suixian county of Hubei Province in the Fall of 1977, and opened in May 1978, has come the richest musical archeological treasure of all time. Among 124 instruments recovered from the tomb are sixty-five bronze bells spanning five octaves, most of them chromatically. Their accurate tuning, wide range, variety of patterns, and 2800 inscribed characters naming tones and identifying keys dramatically alter our notions of early Chinese music. This well-tuned carillon, two thousand years older than any in the West, not only celebrates Chinese virtuosity in bronze casting technology but also establishes her priority in related acoustical physics.

The inscriptions restore many forgotten technical terms to the vocabulary of Chinese musicology. If the bell-chime was originally conceived as a tuning laboratory, as alternate tuning patterns lead some Chinese scholars to believe, the related inscriptions constitute at least a modest musicological library. We now possess both the philological and organological evidence that the emergence of various Chinese schools of philosophy, which Joseph Needham has traced to the fourth and third centuries BC, was preceded

—rather than followed—by a very highly developed musicological structuralism. We can begin to appreciate, as never before, what has always seemed to be an excessive Chinese emphasis on the importance of music.

The bell inscriptions and tuning patterns reveal that the musical thought of ancient China was structurally similar to that of India, Greece, and Babylon. We are now in a better position to study how a fusion of number, tone, and calendar in the third millennium BC launched an 'harmonic' passion whose force—no less in China than in the West from Pythagoras through Kepler—is not yet exhausted. A common musical heritage stretching across so many boundaries of time, space, and cultural preferences invites explanation not only in terms of history but also on the common ground of biophysics.

As a tuned carillon the Zeng chimes predate a comparable Western technology by two thousand years. Coincidences between the tone ratios known to have been used in China and the actual vibrating frequencies of the largest Zeng bells suggest an ability to make accurate frequency determinations, thus casting doubt on the priority of one of our proud achievements in mathematical physics during the last few centuries. At the very least, the musical and technological implications of the Zeng chime set will inspire a new occidental respect for the clarity of the ancient Chinese mind and for the calibre of ancient Chinese science.

Much of the interesting technical information about this archeological event is still available only in Chinese, or remains hidden within the tuning patterns of the bells. The report which follows, made possible by the collaboration of many colleagues and friends, is intended to alert the scientific community to the most novel aspects and to their possible significance. It should be read as a progress report on a continuing study of great difficulty, involving several disciplines, hindered both by the language barrier and by the current inaccessibility of important physical data.

The Tomb of the Marquis of Zeng

Marquis Yi and his small state of Zeng, in what is now Hubei Province, west of Nanjing, were unknown until his tomb was opened in the Spring of 1978 to reveal their names liberally engraved in gold on its artifacts. The date of his interment, c. 433 BC, has been inferred from the inscriptions on a large bronze bell, a funerary gift from his patron, Prince Hui Wang of Chu, which will be discussed in detail later. The four room 'underground palace', covering 2400 square feet, yielded some 7000 burial objects; among them were two orchestras with a total of 124 instruments, covered with some 2800 gold-inlaid characters, many of them concerned with tone names and the transposition system. Although the tomb was filled with water when opened, the carefully lacquered wooden bodies of pipes, drums, and string instruments were preserved, providing examples of certain designs a thousand years older than any known to the museums; only strings and drumheads had disappeared.

In the 'bedroom' with the massive bronze coffin of the Marquis—its outer frame is estimated to weigh eleven tons—was a ten-piece 'silk and bamboo' chamber orchestra of winds, strings, and percussion. In the central 'dining hall' was a larger orchestra of 114 pieces including winds, strings, drums, stone chimes, and bronze bells. Two sets of panpipes have thirteen bamboo pipes arranged in descending scale order; eight pipes of one set can still be sounded. Five *sheng* (a Chinese mouth-organ of bamboo and calabash) with twelve, fourteen, and sixteen pipes still show a special material on the reeds for fine tuning. Seven *se* (a twenty-five string zither, played in a horizontal position)

were accompanied by a collection of thirteen-hundred wooden bridges of some eleven different designs. A set of thirty-two *bian-quing* ('stone chimes') had only nine unbroken *qing*; nearby was a form-fitting wooden case for the stones—so accurately made and carefully labelled that the entire set has now been restored to playing condition. A variety of flutes, drums, and other string instruments included designs unknown before.

Overshadowing everything else in splendor and significance was a set of sixty-five apparently perfectly preserved bronze chime-bells, ranging up to five feet in height, displayed on a magnificent L-shaped frame some nine feet high and thirty-five feet long (3.35 m on the south wall and 7.48 m on the west). Nearby were the coffins of twenty-one young women, ages fifteen to twenty-five—presumably concubines, singers, and dancers—whose skeletons are reported to show no sign of violence.

The sixty-five bell Zeng carillon

The Zeng bells are clapperless—six T-shaped metal hammers and two long heavy rods were found nearby—and are of an unusual 'ellipsoid' design, not well understood, which gives each bell two different fundamental tones, depending on where it is struck. The sixty-five bells thus produce 130 tones.

The pitches of the bells span five octaves, most of them chromatically, in the frequency range between 65 cps and 2330 cps (two octaves below middle C to three octaves above). Most of these tones lie within twenty *cents*, one-fifth of a semitone, of the philosophical pitch standard (middle C = 256 cps), slightly below our international modern standard of A = 440. Many of the individual octaves, fifths, and thirds are as accurate as we could tune them today. The bell now known as number 50 carries an inscription identifying it as *gong* (do) of the Zeng system, and it sounds middle C at 256.4 cps.

The bells in each of the nine sections of the frame are arranged from left to right in descending pitch order except in the bottom row, where their large size and certain revisions of the original plan caused some shifting from the intended positions, known because pitch names were engraved on the frame before the bell order was changed. The pitch range of each set is designated in Table 1 with reference to middle C as C4. The bells belong to seven distinct sets, as will be explained later, tuned in slightly different patterns; this suggests to Chinese scholars that the collection may have been conceived as an acoustical laboratory for displaying alternate temperament systems (Wang, 1981).

In the middle of the bottom row of the chime set is one anomalous bell whose inscription of thirty-one glyphs identifies it as a gift from the patron Prince Hui of Chu:

In the fifty-sixth year of the King, on returning from Xiyang, the King of Chu, Xiongzhang, had ritual vessels made for Marquis Yi of Zeng, to be placed at Xiyang for eternal use and enjoyment (Ma, 1980).

Now Chinese history records the finding of two similar bells at An-lu, forty miles to the south, during the Song dynasty (960–1279 AD). Before these bells were lost again during the Yuan dynasty (1206–1638 AD) their inscriptions—thirty-one characters identical with that of the 'gift' bell—were copied and preserved. Chu-hui Wang was the Duke of Chu between 489 and 432 BC, and the fifty-sixth year of his reign was 433 BC (Lee, 1980). It is assumed that the tomb of the Marquis of Zeng would have been sealed not long afterward.

The bells and their stand are covered with gold inscribed characters naming the pitches—the two lower rows in the key of C—according to a twelve-tone relative pitch system

Table 1. Physical arrangement of the Zeng chime bells. The thirteen large bells in the bottom row are no longer in their original order, but all of the others in each set are arranged from left to right in descending pitch order. Sets Ib and Ic belong together, and so do IIb and IIc, thus there are only seven separate sets

South Wall	West Wall	
Set Ia 6 'descant' bells D7 to C5	Set Ib 6 'descant' bells Ab6 to Ab4	Set Ic 7 'descant' bells A7 to Gb4
Set IIa 11 'soprano' bells C7 to D4	Set IIb 12 'soprano' bells C7 to D4	Set IIc 10 'alto' bells C6 to G3
Set IIIa 3 'bass' bells	Set IIIb 6 'tenor' bells	Set IIIc 4 'tenor' bells

never encountered before. Eight of the twelve names emphasize the importance of the major third, a characteristic of 'Just' tuning. It had heretofore been assumed that the spiral of fifths was the only tuning ever used in China. The frequencies of the bells confirms that the makers may very well have had in mind the ancient 'Just' system which inspired the cosmologies of many ancient civilizations (McClain, 1976; 1978). Several bells carry additional inscriptions identifying tones also according to an 'absolute pitch' system in the key of A-flat, using for this purpose the twelve traditional Chinese tone names.

The existence of a 'fixed' or 'absolute' pitch system has never before been documented anywhere in antiquity, although Chinese interest in the subject is well known. The whole Zeng naming and tuning structure proves to have an impressive internal logic. And although bronze bells have been known throughout Chinese history (examples go back to the Shang dynasty in the fourteenth century BC), until the Zeng bells were found—the first to have pitch names engraved on the striking points—it was not understood that this (Zhou) design gave two fundamental pitches. The largest set of Chinese bells previously known had been a set of thirteen excavated in 1957, now preserved in Chengchow; most of the earlier sets in various museum collections contain only two, three, five, or eight bells. In 1977 a set of five large and eighteen small bells were found in the tomb of Fu Hao, one of the sixty-four wives of the Shang emperor Wu Ting in the twelfth century BC, but they are of an earlier design (Chang, 1980).

The largest Zeng bell, sounding the lowest pitch, two octaves below middle C, is almost five feet high and weighs about four hundred fifty pounds. To gain some initial perspective on its significance it helps to remember that the largest bell known in the West this early in history is an Assyrian harness bell about four inches high. Roman bells of the first two centuries AD reached eight inches, the size of the smallest Zeng bell, but by 1000 AD 'no bell more than two feet high had been seen or heard of in Europe' (Needham, 1962).

After being taken to Beijing for extensive tests by the National Music Research Institute, the Academia Sinica, and scholars from the universities, and being heard in a radio broadcast, the Zeng bells were returned to the Hubei provincial museum in Wuhan, where they now entertain visitors.

Part 2: Technological considerations

Ancient Chinese bell designs

Ma Cheng-yuan describes Shang and Zhou bells of the first millenium B.C. as flattened ovals whose two sides meet in a fairly sharp angle at the seam. The thick edges of these two vertical side seams somewhat restrict the vibrations of the front and back plates, he suggests, reflecting vibrational waves rather than allowing them to rotate around the perimeters as freely as they do in our familiar circular bells (Ma, 1980). These side seams are extended downward, giving the bell a pair of 'ears' and a high waist so that the mouth is neither round nor flat. When struck on the middle of this high waist—the *sui* striking point, reported to produce the greater volume and lower pitch—waist and end points alternatively contract and expand, as suggested in Fig. 1(a). When struck midway between this *sui* point and either end seam—at the so-called *gupang* or *gu* striking points, producing a higher pitch and sometimes weaker volumes—as shown in Fig. 1(b), both *sui* points and the end points now become stationary nodes. (The strike point is always an antinode, a place of maximum displacement, in the vibratory motion of the fundamental mode.)

Ma has published holographic photographs of four similar small bells now in the Shanghai museum which show the typical nodal lines and circles associated with bell partials, and the end seams are antinodes for some of these partials. Until a more extensive report on the physical measurements and acoustical spectra of the Zeng bells is available, describing both onset and decay characteristics, noting evidence of filing after coming from the mold, assessing the probability of each bell now being in its

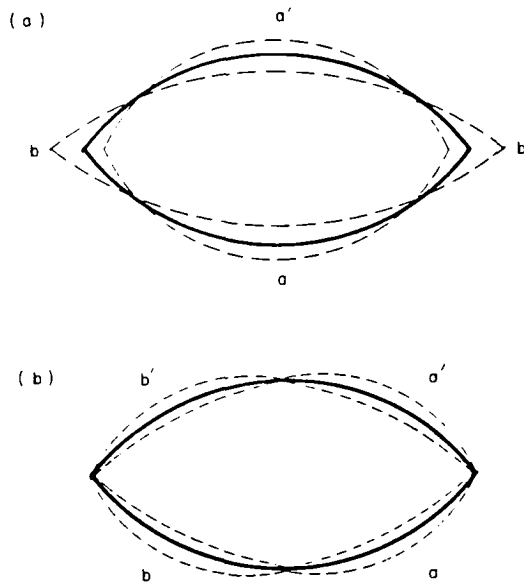


Fig. 1. Alternate vibrational modes. Ancient Chinese bells have two different fundamental modes of vibration. When struck at point *a* in Fig. 1(a), sides and ends of the oval alternately contract and expand (i.e. they are antinodes in the wave pattern). When struck at point *a'* or *b'* in Fig. 1(b), these contractions and expansions occur nearer to the end points, so that the antinodes in the first example become nodes (places of least motion) in the second. The pitch of this second fundamental tone is from one to four semitones higher. (By permission of Chinese Music.).

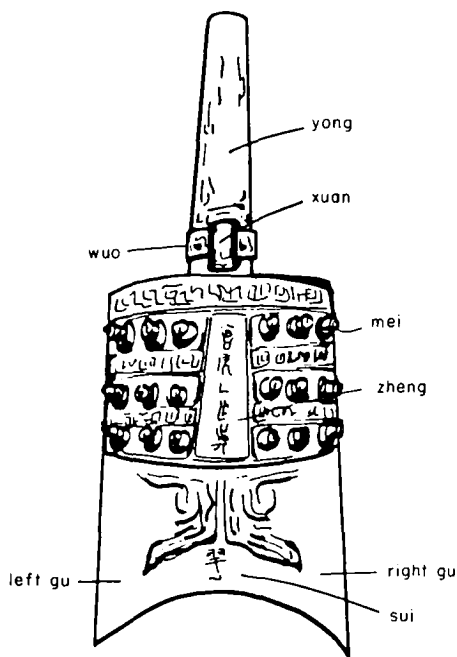


Fig. 2. Chinese yongzhong bell design. The sui striking point sounds the lower fundamental, and the right and left gu striking points sound the upper fundamental. Notice the protruding mei 'nipples'. Small bells were carried by the yong 'nose' shanks, mouth upwards. Larger bells, like the forty-five in the lower two rows of the chime set, were suspended by a ring (xuan) on the waistband (wuo) of the shank. (By permission of Chinese Music).

original physical condition, and, hopefully, accompanied by high fidelity recordings of each separate tone, explanations of how the bells actually behave remain somewhat conjectural. They are a design phenomena outside the experience of western physicists.

The Zeng bells show three variations of the basic design:

- (1) The nineteen small *niuzhong* bells in the upper row have short *niu* 'hanging loops' cast into the shoulders of the bells. These loops are inserted into holes made in the bottom of the supporting beams, and are then locked into place by copper bolts inserted through side holes.
- (2) The large *yongzhong* bells in the lower two rows have long *yong* 'nose' shanks, as shown in Fig. 2. The bell design evolved from a grain scoop: this 'nose' is the handle by which small bells were carried inverted, mouth upward. These large Zeng bells, however, are suspended by a ring (*xuan*) on the waistband (*wuo*) of the shanks from hooks mounted on the beams of the chime frame; their mouths hang downward at an angle of about thirty degrees from perpendicular, the weight of the bells being partially counter-balanced by the *yong* shanks.
- (3) The 'gift' bell in the bottom row is a *bo* design, being a flattened oval like the others, but with a hanging loop like the small *niuzhong*. It lacks the 'ear-like' extensions near its side so that its mouth, while oval, is also flat. This bell was apparently not cast in Zeng (it lacks the elaborate identifying inscription), and its upper pitch is reported as weak and uncertain.

Of the forty-five *yongzhong* in the lower two rows, all but twelve bells of the middle set in the middle row (labelled set IIb here) have three rows of *mei* 'nipple' protuberances cast into the bell circumference (see Fig. 2). These protuberances are now thought to be local 'loading' devices: 'with the mei design, only selected overtones of each fundamental are present in music making, and the two sets of vibration modes are more easily separable'. If indeed they function as initially described, they are an 'incredible acoustical design', as *Chinese Music* claims, and Western physicists are interested in learning more about them. A slight thinning of the waist in the *gu* striking areas and the 'ear' extensions at the seams are also thought to be efforts to make the upper fundamental tone as strong and clear as the lower one. Since a replication of the entire set of Zeng chime bells had its premiere concert in Beijing on 25 October 1984, we must assume that Chinese scientists have now learned much more about the Zeng design and will publish their findings in due time (*Scientific American*, 1984).

The control of pitch and timbre

Producing a set of bronze bells tuned accurately to the scale—beyond Western technology until the fifteenth or sixteenth centuries—poses special problems. Pitch can normally be altered very little once the bell comes from its mold. Success depends not only on accurate models from which to build the clay molds but also—in order to achieve the geometry necessary for a particular pitch—on a meticulously controlled molding and casting procedure. A twentieth-century American chime manufacturer, for instance, is reported to have cast two or three bells of each size, selecting the best candidate for fine-tuning, apparently limited to within a fraction of a semitone, and then selling the others separately. Some of the Zeng bells show evidence of abrasion on the inside of their rims, but we may doubt that this afforded their makers very much control over pitch.

Needham and Robinson, quoting extensively from an early treatise on bell making, note that the copper was melted three times before casting to ensure its purity, and that the proportions of copper and tin were carefully measured in standard vessels which

could themselves be checked by the pitch of the notes they gave when they were struck, as it was contrived that they should emit particular notes of the scale (Needham, 1962).

How far this integration of grain scoops, standard measures, and bell design was carried will be noted later.

With our western round bell design it is possible to 'scale' a carillon by making all dimensions proportional; if the ratio of thickness to diameter is held constant, 'frequency varies as the inverse of the diameter'. According to Rossing (1984), 'this method, known as $1/f$ scaling, approximates that found in many carillons dating from the fifteenth and sixteenth centuries . . . when the art of making bells started to flourish in the Low countries'. It was necessary to increase the size of small treble bells; however, to counteract their tendency to give a rather weak sound. The smaller Zeng bells have problematic tunings. The clay 'piece-molds' for the larger and better tuned Zeng bells are estimated to have been assembled from as many as 137 different sections. So expertly was this process controlled that half of the bells show the same ratios in their various dimensions, and the others do not vary by more than four to eight percent (Hua & Jia, 1982). Since copper furnaces in that period are known to have reached a diameter of 60 to 80 centimeters, capable of melting 100 kg of copper in one firing, it is believed that the contents of two such furnaces were probably sufficient for a large bell, for the large Zeng low C weighs only 203.6 kg. It is obvious that the remarkably accurate tunings in the

Zeng set must be attributed to a virtuoso bronze casting technology, and to astute awareness of the principles of design.

Early Shang bells often have a minor or major second between their two fundamentals; later the minor third becomes more frequent. Among the Zeng bells, forty-seven possess minor thirds and sixteen have major thirds. The higher pitch is never a harmonic of the lower one; it is the result of a quite proper second fundamental mode of vibration, each fundamental enjoying its own characteristic spectrum of higher partials.

The ellipsoid shape, with its heavy side seams, is credited also with attenuating the tone to some extent, thus rendering the bells more useful for a melodic carillon. The tone of the bells has been described as 'sweet and beautiful' (Hua & Jia), the middle row 'melodious and resonant', the lower row 'deep and sonorous' (Quian *et al.*, 1981). Cassette recordings made from Beijing radio performances justify these adjectives and suggest that the bell fundamentals, unlike those in our typical Western bells, ring with an unusual clarity. They are less affected by the high partials, usually inharmonic, which give our church bells a strong component of 'noise' and complicates our perception of their pitch.

The 'strike-tone' of the round bells in our Western carillons is normally a 'subjective' pitch inferred by the ear from a cluster of higher partials which have been very subtly tuned; the bells are mounted on a lathe and turned to a thickness—determined by an empirical formula which considers the effect on each partial—which makes several partials approximate an harmonic series (Rossing, 1984). This 'strike tone' is generally an octave above the 'hum-tone' of the fundamental. Because of the differences in design, the Zeng bells are able to sound a true fundamental, and to do so with only one percent (*sic* 1/100) of the weight our bells require. The largest Zeng bell, for instance, needs only 450 pounds of bronze to sound C2 two octaves below middle C. To produce the same pitch as the 'strike tone' on a typical round bell requires between 40,000 and 50,000 pounds of bronze, with attendant complications at every step of the casting and tuning process. The entire Zeng chime-set weighs less than five tons.

Part 3: Musicological significance

Zeng relative pitch names

All of the chime bells except the 'gift' bell and the incomplete set (1a) of six small bells in the top row carry elaborate, gold-inlaid inscriptions identifying them as made in the state of Zeng. All bells except the 'gift' bell also carry engraved names in a twelve-tone relative pitch system. This pattern of names, because of its emphasis on the major third, suggests a 'Just' tuning system (meaning one with some pure thirds of 5 : 4) never before documented in China. The naming system assigns single character names to four basic tones from the spiral of fifths: *gong* (or *kung*) do, *zhi* (or *chih*) sol, *shang* re, and *yü* la, corresponding respectively to C G D A. These four names are then repeated with a second character, *jue* (or *chiao*, or *jiao*, depending on romanization preferences) for four more tones a major third higher on E, B, F-sharp, and C-sharp, and with another character, *zeng*, for four tones a major third lower on A-flat, E-flat, B-flat and F. This triple use of four basic names at the interval of a major third constitutes philological evidence that such thirds were thought of as entities in their own right, rather than merely as tones indirectly related through the spiral of fifths. The tunings for these tones carrying double character names confirm that most of them were probably conceived as 'pure' thirds of ratio 5 : 4, thus in a tuning pattern now known as 'Just'. (Spiral fifths

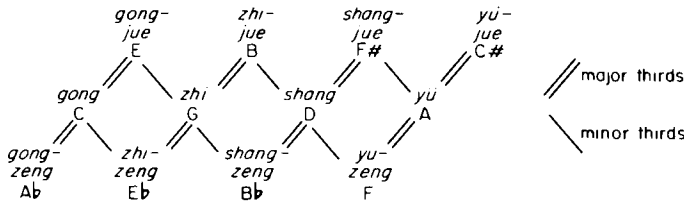


Fig. 3. The 'Just' pattern of Li Chunyi. Tones in each row belong to a spiral of fifths, but the rows are related by 'Just' major thirds of 5 : 4 along the right diagonals /// and by their minor third complements within the fifth along the opposite diagonals \\\\. Note the triple use of four basic names, gong, zhi, shang, and yu.

produce larger thirds of 81 : 64.) To make clear the interrelations implied in these unusual pitch names, Li Chunyi presents the graph in Fig. 3; I merely substitute romanizations and English tone names. Reading from left to right, all rows show sequences of musical fifths; reading upward and downward along the diagonals, ///, all intervals are major thirds. Along the opposite diagonals, \\\, all intervals are minor thirds.

The forty-five large bells in the bottom two rows are named in our modern key of C. The thirteen small bells in sets IIb and IIc are named in the key of G-flat (being treated here as equivalent to F-sharp). The first six small bells in set Ia, however, apparently not made in Zeng, and not forming a complete scale, seem to be named in two different keys, G-flat and E-flat (three bells each), and their tuning seems erratic.

We are ready now for the interesting problem of trying to deduce from the measured bell frequencies exactly what tuning pattern the makers intended, and how accurately they achieved it.

The one-hundred thirty tones

Bell tunings were measured by Huang Xiangpeng, chairman of the research committee entrusted with their study, and by Wang Xiang, and were published by Wang in *Yinyue Yanjiu* 1981. They employed a strobotuner graduated to read plus or minus deviations of up to fifty cents, a cent being one-hundredth of a semitone, from our modern equal temperament standard of A440 cps. Temperature is reported as 30–32 degrees centigrade. The tones are shown in Fig. 4 in the order in which the bells are arranged on their frame.

Wang observed that the average pitch of the thirty-three main melody bells in the second row is 38.7 cents below our A440 standard, very close to the 'philosophical pitch' which places middle C at 256 cps, and in agreement with the frequency of bell number 50 (256.4) which is labelled gong 'do' of the Zeng system. I have accordingly raised all of Wang's published readings by thirty-eight cents to effect a conversion from A440 to C256 cps. (At this new pitch level, A is reduced to 430.5 cps.) Plus and minus cents deviations are now distributed symmetrically, facilitating analysis of the tuning patterns.

Considering the difficulty of designing and casting a tuned carillon in any age, with the additional complication of having to engineer not one but two useable pitches, and considering also the uncertain effects of corrosion over twenty-five centuries, and perhaps further effects from cleaning, and mindful of small errors necessarily involved not only in the testing process itself but also in any original 'monochord' model for the system, it would have surprised no one if the Zeng bells had displayed merely a random

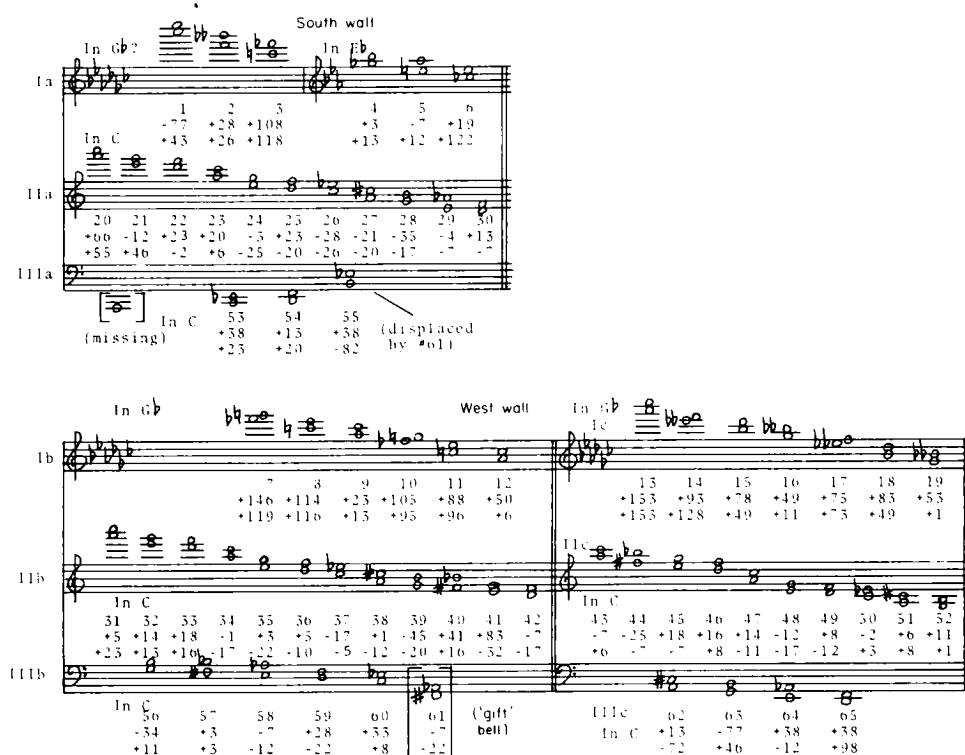


Fig. 4. The Bronze chime Bells of Zeng. Strobotuner readings are given as cents deviation from equal temperament at C = 256 cps. Set Ia is problematic and incomplete, it appears to be named in Gb and Eb. Sets Ib and Ic are named in Gb. The bells on the lower two rows are all named in C.

distribution of pitches. Even a cursory inspection of the cents readings in Fig. 4, however, shows that over three-quarters of the bell tones in the lower two rows (sixty-nine out of ninety) lie within one-quarter of a semitone (i.e. 25 logarithmic cents) of our modern equal temperament, a range of discrepancy barely within the threshold of perception. Taken by themselves, out of context in their sets, the very best octaves and fifths among the bells are so perfect that they could not be improved today. A few worst examples are so obviously wrong that we should probably assume either an error in labelling or deterioration in the tomb and eliminate them from consideration.

The equal temperament built into our measuring instruments is a 'mean' between the ancient 'spiral of fifths' and its 'Just' counterpart, the only tuning systems we can plausibly attribute to China. To get a better idea of Zeng accuracy we must adjust our cents reading to compensate for musical fifths of 3 : 2 being 702 cents (not 700), and for 'pure' major thirds of ratio 5 : 4 being only 386 cents (not 400). Figure 5 uses Li Chunyi's pattern to display how a perfect 'Just' tuning and a perfect 'spiral of fifths' tuning would look to our strobotuner if both systems were judged from the perspective of C at the 'philosophical' pitch level. Middle C would then vibrate at 256 cps and show a deviation of zero cents.

That we are viewing the tones in Fig. 4 from the proper pitch standard can be verified by computing their mean deviations from these two alternate systems. In the lower two rows of the chime set, discounting five tones which are obviously mislabelled, the mean

e	b	f#	c#
C	G	D	A
ab	eb	bb	f

-14	-12	-10	-8		+8	+10	+12	+14
0	+2	+4	+6		0	+2	+4	+6
+14	+16	+18	+20		-8	-6	-4	-2

'Just' on C		'Spil' of fifths' on Ab
(both measured from C 256 = 0)		

Fig. 5. Conversion tables for strobotuner readings. Modern strobotuners, calibrated to equal temperament, must be corrected by the amounts shown above for 'Just' and 'spiral of fifths' tuning. The four basic tones C G D A, given single character names, belong to both systems, but eight others are sharp or flat by larger amounts, depending on which tuning procedure is employed (note the difference of twenty-two cents between them). The cents readings in Fig. 4 must thus be corrected by this table to give any fair idea of the accuracy of Zeng tuning.

deviation of the other eighty-five from either 'Just' tuning or 'spiral of fifths' tuning is less than one cent. Thus 'philosophical' pitch is securely established as the appropriate reference standard for our measuring instruments, no matter how the ancient Chinese thought of it or how they achieved it. What they are likely to have had in mind will be deduced later. (I have eliminated from consideration the lower tones on bells numbered 55, 62, and 65, and the upper tones on 41 and 63.)

Notice that in Fig. 5 four tones, C G D A, are common to both systems and therefore cannot be used to differentiate them. But the upper thirds (E, B, F-sharp and C-sharp) which are flat in 'Just' tuning are sharp in the 'spiral of fifths', while the lower thirds (A-flat, E-flat, B-flat, and F) show the opposite changes. These eight tones each show twenty-two cents difference between the two systems. Among the eighty-five tones which we are considering, the common tones C G D A occur forty-one times with a mean deviation of only 0.12 cents and a standard deviation (calculated by the method of 'least squares') of twenty-one cents, that is, about one-fifth of a semitone, subliminal under most conditions. A study of the standard deviations of the other forty-four tones, both as a group and within their separate sets, strongly supports the hypothesis that they were conceived as 'Just' thirds of 5 : 4. The eight-five tones of the combined sets show a mean deviation from a perfect 'Just' tuning at C 256 of only 0.5 cents, and a standard deviation of only 21 cents, an accuracy astonishing to anyone familiar with bells. Today the best piano tuners, working under infinitely easier conditions, show standard deviations of up to ten cents from their equal temperament standard.

Wang Xiang, however, offers a group of hypotheses to support a quite different view of the bell tunings. He considers only the ten bells of set IIc to illustrate the 'Just' hypothesis, and then introduces a series of *ad hoc* assumptions to save three other sets, which become sharper in their higher registers, for the 'spiral of fifths' point of view. In the nineteen small bells in the top row, he suggests, the familiar 'comma' of twenty-four cents in the spiral of fifths (each perfect fifth is two cents too large for perfectly cyclic

tuning) has been allowed to accumulate through three octaves. (The appropriate Zeng figure, he notes, is actually twenty-three cents per octave.) Set IIa, he proposes has a cumulative comma of thirty cents in each octave, and set IIb a cumulative comma of fifteen cents. The virtue of Wang's theory is that it fits the measured frequencies of each set of bells well enough to allow him to 'save the appearance' (in the Platonic sense) for the traditional Chinese spiral of fifths, while claiming for Zeng acoustical science the possible intent to experiment with different systems of temperament.

The difficulties with Wang's theory are its lack of generality (a different hypothesis for each set), the absence of any reason or tradition for letting commas of 15, 23, and 30 cents accumulate across several octaves, the extreme difficulty of computing such progressions, today made easy by logarithms (he assumes the bells were tuned by ear), and a general 'dissonance' with ancient tonal and numerical procedures. It seems doubtful that so meticulous a casting procedure would have lacked a mathematical tuning model.

The issue as to which tuning system was intended in Zeng is critical for a view of Chinese history which will be developed later. I am more inclined to suppose that some design or casting problem (or even a corrosion and cleaning problem) may be responsible for a general bias toward sharpness in the higher bells (not true, however, for set IIc or the large bells in the bottom row). But Wang's theory is ingenious, and I am much indebted to his careful study of the tuning patterns. I cannot disguise my own preference for the simpler 'Just' hypothesis offered earlier, particularly as it is in agreement with the naming system on the bells, accessible by the simplest arithmetic procedures, and consonant with so many other factors yet to be discussed. Whatever the truth of Zeng intentions, and future archeological findings could teach us more, Zeng acoustical science deserves universal admiration.

The small bells in the top row

The nineteen small bells in the top row of the chime set pose several problems. The first group of six, in set Ia, lacking the Zeng identifying characters, have engraved tone-names which indicate that all were intended to have major thirds between their two fundamentals. Sets Ib and Ic (see Table 2), however, all named in the key of G-flat, have minor thirds between their fundamentals; both of these sets become sharp in their higher octaves. Taken together these three sets again emphasize the defining aspect of 'Just' tuning—the important role of the major third and its minor complement.

Tan Weisi and Feng Guangsheng develop the argument that the thirteen bells in sets Ib and Ic were once unified in scale order, and were suspended from the middle beam on which set IIa now hangs. They noticed that in the underside of this middle beam on the south wall, a series of fourteen holes had been made—with corresponding side holes—like those from which the small *niuzhong* in the top row are now suspended. These holes had been plugged with wood and painted over, but one plug had fallen out, and the spacing between the other holes, now showing through the paint, can be measured. These holes are spaced appropriately to accommodate all thirteen bells in sets Ib and Ic, plus one additional bell now missing. Supporting this argument is the fact that the eleven larger *yongzhong* now occupying this section of the frame actually overflow its bounds so that two bells are suspended from the very end, to the detriment of the aesthetic effect obviously intended. The present arrangement of the whole top row of the chime set thus appears to be an afterthought. Tan and Feng re-unite the thirteen bells of sets Ib and Ic in scale order and identify the fourteenth bell, now missing, as the upper octave of number 7, presumably sounding the high G-flat 6 and B-flat 6; it would be the second

Table 2. The organization of sets Ib and Ic. The pattern of names and sounds reveals that bell 13 belongs ahead of bell 7. The vertical columns progress by major thirds, the tones on each line are minor thirds, thus the pattern emphasizes the unique aspects of 'Just' tuning. In the 'two-element' theory, apparently developed later, the left column contains yang male tones and the right column contains yin female tones

Number	Lower fundamental (yang tones)		Upper fundamental (yin tones)	
		Set Ib		
7	shang-zeng	Fb6	yü-jue	G6
8	shang-jue	C6	yü	Eb6
9	shang	Ab5	yü-zeng	Cb5
10	shang-zeng	Fb5	yü-jue	G5
11	shang-jue	C5	yü	Eb5
12	shang	Ab4	yü-zeng	Cb4
		Set Ic		
13	shang	Ab6	yü-zeng	Cb6
14	gong-zeng	Ebb6	zhi-jue	F6
15	gong-jue	Bb5	zhi	Db6
16	gong	Gb5	zhi-zeng	Bbb5
17	gong-zeng	Ebb/D5	zhi-jue	F5
18	gong-jue	Bb4	zhi	Db5
19	gong	Gb4	zhi-zeng	Bbb4

highest bell of the combined set, and it properly deserves the place now occupied by bell number 13 at the head of set Ic, as shown here in Fig. 6.

The relative pitch tone-names in G-flat/F-sharp engraved in gold on the strike-points of the bells in sets Ib and Ic exemplify the symmetric 'Just' schema of Fig. 3. If bell 13 were moved ahead of number 7, where it obviously belongs, both sets of bells would then progress by major thirds through two octaves, with the yang male tones on the lower fundamentals and the yin female tones on the upper ones. This appears to be the most extensive of early examples, albeit an implicit one, of such a segregation by 'sex', and in the century before Needham thinks this theory of 'two fundamental forces' became a central tenet in Chinese philosophy. The pattern emphasizes the basic role of the four names, *gong*, *shang*, *zhi*, and *yü*, and the importance of the major third. It is difficult to avoid the conclusion that a specifically didactic intention is at work in this top row; the physical arrangement of the bells calls attention to the 'Just' tonal organization. An even more significant purpose will be suggested later.

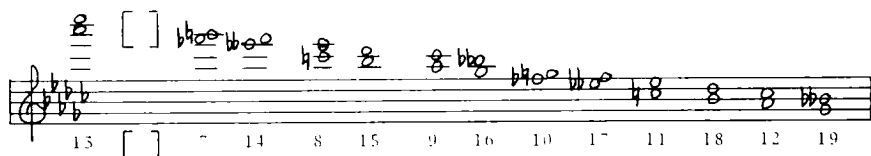


Fig. 6. The combined tones of sets Ib and Ic. These two sets appear to have been originally unified; one bell is now missing. Chinese tone-names place the tonic (gong) on G flat. Note that bell 13 belongs ahead of 7.

Musical implications of the Chime scale

In the absence of any musical examples preserved from ancient times, the rich variety of tonal perspectives displayed on the various Zeng chime sets is both illuminating and provocative. Huang Xiangpeng points out that among the forty-five bells producing ninety tones in the key of C, the perfect fourth on F-natural occurs eleven times while the tritone fourth on F-sharp—formerly considered normative in ancient times because it occurs as the seventh tone in a spiral of fifths (i.e. C G D A E B F#)—occurs only four times. To Huang this means that the standard form of the ancient diatonic scale, long considered the ‘new’ scale in China and thought to have been introduced much later by the Mongols, is proved to be our own familiar C major scale. The tones B and F-sharp traditionally have been named as lower auxiliaries of the C and G a semitone above; but here in Zeng this fourth step on F-natural already has a name of its own, *he*. Huang also emphasizes that although the tones are named in the key of C, the chimes themselves seem to favor D as the modal tonic; the lowest tone in sets IIa, IIb, and IIc is *shang* re D. He also considers the distribution of bell pitches to favor D, G, and C as the primary structural tones. The fact that the three deepest bass bells were A (now missing), C and D may also say something about the general significance of these three tones.

To anyone familiar with ancient Greek musical theory and the Babylonian ‘musical proportion’ on which it is grounded—6:8:9:12, still the basis of all Western tonal music (tonic—subdominant—dominant—tonic octave)—the wealth of such progressions among the Zeng bells is striking. The interlocked fourths and fifths of this tonal framework show a powerful internal, psychological bias toward symmetry, one not to be explained acoustically at all for the perfect fourth above the tonic does not belong to the arithmetic sequence derived from the ground-tone, thus it is not part of the harmonic overtone series (hence not inspired by hearing it subliminally). The presence of a perfect fourth 4:3 above the tonic is the purest evidence of an internal, human artistic bias in favor of tonal ‘balance’, born of memory, mother of all of the arts. Sets IIa and IIb in Fig. 4 flaunt these progressions fully exposed in their highest two octaves, A–D–E–A and C–F–G–C, and all three sets in the middle row reaffirm the pattern among their lower tones.

Every ancient Greek scale from Archytas through Plato, Ptolemy and Aristides is developed on this same frame. The coincidence here is at least as surprising—later Chinese constructions are dominated by arithmetic convenience alone—as the discovery that the normative diatonic scale is our modern C major scale pattern, and at the ‘philosophical’ pitch level long used in organ building. The bells in fact present themselves in a greater variety of tonal and modal perspectives, including pentatonic, hexatonic, and heptatonic progressions, than could ever have been imagined from the single pentatonic tuning pattern—C G D A E—which is all that literary records preserve from this early period in Chinese history. Not only is the Zeng bell technology astounding, so also is the evidence of a rich tonal development with so many structural parallels to the West, in a culture whose musical styles are unique. Our common human biological bias could hardly be clearer, for what is old and universal could not endure unless continually revalidated in human nature.

The coincidences between East and West noted here prompt with new urgency the old question of a possible common Babylonian origin. The spiral of fifths tuning, long thought to be the only one ever used in China, has recently been traced to Babylon in the eighteenth century BC (Kilmer *et al.*, 1976). Chinese mythology preserves the tale of a journey to the western mountains c. 2800 BC to find appropriate bamboo for the pitchpipes; an elaborately gold-ornamented lyre and two silver pipes entombed at Ur

c. 2800 BC suggest that at that time Mesopotamia actually enjoyed a high development of instruments capable of teaching tone ratios. Later Greek mythology preserves the tale of Pythagoras' journey to Babylon to learn the 'musical proportion' 6:8::9:12 which we now know was fundamental in both Greece and China. In the Semitic culture of ancient Babylon, the gods themselves were musicalized with numerical 'nicknames' whose logical rationale is found in the musical scale (McClain, 1982; Stieglitz, 1982). The great antiquity of a high culture in Mesopotamia and its many coincidences with later developments East and West suggest that it radiated a basic psychophysics of tone and number in both directions. Vedic India acquired this material early enough to have been a conduit (de Nicolás, 1976; 1982; Larson, 1982).

Evidence for an 'absolute' pitch system

The chime inscriptions leave no doubt that Zeng identified absolute pitch by the twelve traditional tone names generated from a *Huang-chung* fundamental on A-flat. Table 3 presents these names, which presumably date from at least as early as the sixth century BC. Months follow the rising chromatic order; *Huang-chung* is assigned to the eleventh month quite possibly because the computation for a spiral of twelve musical fifths must begin on $3^{11} = 177,147$ to avoid fractions (McClain & Hung, 1985).

The evidence begins with bell number 50 named *gong* (do) and sounding middle C; the pattern of names and pitches on the other forty-four *yongzhong* in the lower two rows confirm that C is indeed the tonic of the relative pitch system. But this bell also carries an additional inscription as 'gong of *gu-xian*' mode, thus identifying C with the tone *gu-xi* or *ku-shi*, a major third above *Huang-chung*, which consequently must have been A-flat (see Table 3). This initial and indirect identification of A-flat as *Huang-chung* is reaffirmed both directly and indirectly by inscriptions on several small bells in sets Ib and Ic in the top row, a group which is itself tuned to the key of G-flat/F-sharp. Down through history the only Chinese terms for describing exact intervals remained that of counting consecutive 'chromatic months' (Nakeseko, 1957).

Table 3. Absolute pitch names in A-flat. The twelve tone names shown here have a fixed relationship with *Huang-chung* so that musical intervals are conveniently designated by the number of months between them in the chromatic order. *Ku-shi*, for instance, a major third higher, is four chromatic tone-months later. The series is traditionally conceived as a spiral of fifths generated from *Huang-chung*, but for reasons explained in the text and in the following figure, the pentatonic scale on C also has its 'Just' origin on A-flat

Tone	Month	Name	Tone	Month	Name							
Yang tones			Yin tones									
Ab	11	<i>Huang-chung</i>	A	12	<i>Ta-lü</i>							
Bb	1	<i>T'ai-ts'ou</i>	B	2	<i>Chia-chung</i>							
C	3	<i>Ku-shi</i> (gu-xi)	C #	4	<i>Chung-lü</i>							
D	5	<i>Jui-pin</i>	D #/Eb	6	<i>Lin-chung</i>							
E	7	<i>I-tse</i>	E #/F	8	<i>Nan-lü</i>							
F #/Gb	9	<i>Wu-yi</i> (wu-hsieh)	G	10	<i>Ying-chung</i>							
Fifths	1	2	3	4	5	6	7	8	9	10	11	12
Yin		Eb		F		G		A		B		C #
Yang	Ab		Bb		C		D		E		F #/Gb	

Bell number 19 sounding G-flat 4 is named *gong* of sets Ib and Ic; it is also inscribed, “*gong* of *Wu-yi*” (*Wu-hsieh*), thus identifying G-flat as the minor seventh of a set generated from A-flat.

Bell number 18 sounding B-flat 4 and named *gong-jeu*, meaning a major third above the tonic in the key of G-flat, is also labelled ‘*gong* of *T’ai-ts’ou*’, thus identifying B-flat as the major second in a series generated from A-flat.

Bell number 11 sounding C5 and named *shang-jue*, meaning a major third above *shang* A-flat and thus a tritone above the tonic G-flat, is also labelled ‘*gong* of *gu-xi*’ (*ku-shi*), again identifying C as the major third of a series generated from A-flat, and confirming the inscription on bell number 50.

Bell number 12 sounding *shang* A-flat, major second in G-flat, is also inscribed ‘*gong* of *Huang-chung*’, thus explicitly identifying A-flat as the absolute pitch of *Huang-chung*.

Bells numbered 9, 10, 15, 16, and 17 are similarly labelled, leaving no doubt that the absolute reference pitch is always the tone A-flat.

Bell number 25 sounding *shang* D6 carries the additional inscription, ‘the *xia-jue* [major third] of *muzhong*, the *shang* [major second] of *gu-xian*’, which helps Huang to identify the basic *muzhong* mode of the state of Chu as the key of B-flat, a wholetone lower than the *gu-xian* C mode of its vassal Zeng. This interpretation of *muzhong* as B-flat is reinforced by an inscription on bell number 53 in the bottom row, now the largest of all and sounding the low *gong* C2, describing it as ‘*shang* [re] of *muzhong*’ [B-flat] as well as ‘*gong* of *gu-xian*’. This again reinforces the identification of C as a major third above the *Huang-chung* A-flat in the absolute pitch series, noted earlier on bells numbered 50 and 11, by making it a major second above B-flat, while also naming it as tonic (*gong*) in the key of C used for all bells in the bottom row.

Bell number 54, second largest of the Zeng bells and sounding the low *shang* (re) D2, has a whole series of inscriptions linking this pitch with other keys and other states, always by way of the absolute pitch names in A-flat shown in Table 3.

Gong [do] of *Jui-pin* [meaning that D is a tritone above A-flat], also called *Pin-huang* in Chu State and *Ch’ih-tse* in Chen State, same as *gong-jue* of *T’ai-ts’ou* [meaning that D is a major third above B-flat], *gong-zeng* of *Wu-hsieh* [meaning D is also a major third below F-sharp/G-flat], and *shang-jue* of *Huang-chung* [meaning that D is also a tritone above A-flat].

These multiple cross-references between pitches in the two different naming systems prove that Zeng possessed both a twelve-tone absolute system in the ‘calendar’ key of A-flat and a twelve-tone relative system, fully embodied in the keys of C and of F-sharp/G-flat. The structural completeness, with its fully developed terminology, is impressive.

Evidence for frequency determination

An absolute pitch reference system could have been established by arbitrarily taking *any* pipe or bell as the reference standard. But there are correspondences between tones and numbers which logically justify the choice of A-flat as the reference pitch, and which remain hidden behind the cents measurements published by Huang and Wang until their comparative strobotuner readings are converted into frequencies.

The oldest Chinese calculations for the scale, those of Kuan-tzu (d. 645 BC), employ a ‘trisection’ rule which is the simplest ever stated for producing a spiral of fifths on pipes or strings: ‘add or subtract one-third’. The rule produces rising fifths 3:2 or falling fourths 4:3, their octave-complements. To produce five such tones while avoiding

Table 4. Pentatonic frequencies. The deepest bells now sound the pentatonic scale at frequencies closely approximating the ideal ratio numbers. In any other culture this might be casually dismissed as pure coincidence, but the overwhelming structural coincidences involving the Zeng chimes encourage belief that frequencies were contrived to match the ratio numbers

Bell	53	54	64	63	62
Tone	C	D	E	G	A
Ideal Ratios	64	72	81	96	108
Frequencies	64.8	72.6	80.1	98.4	103.2

fractions, a characteristic of musical arithmetic all over the ancient world, it is necessary to begin on the traditional Chinese number $3^4 = 81$ (one factor of 3 for each subsequent operation).

C	G	D	A	E
81	108	72	96	64
+ 1/3	- 1/3	+ 1/3	- 1/3	

Kuan-tzu carefully pointed out that 81 contains four factors of 3. Now frequency ratios are the inverse of pipe or string ratios, and so no matter what the actual pitches may be, the abstract frequency *ratios* for this pentatonic group in the normative scale arrangement (rising C D E G A) will begin on 64 and end on 81, as shown in Table 4. Notice how remarkably close the *measured* frequencies of the lowest Zeng pentatonic tones (readily converted from the cents measurements in Fig. 3) approach these *abstract* frequency ratios. The difference between the ratio number 64 and the measured frequency number 64.8 is minimal; so are the differences between 72 and 72.6, 81 and 80.1. We are looking here at a remarkable historical anomaly: the abstract ratios for this scale coincide with the actual frequencies on a set of bells cast more than two thousand years before Western scientists learned to make accurate frequency counts. Notice that if the intention was to produce a set of five bells vibrating at the integer ratios shown above, to us it would inevitably appear as 'philosophical pitch' (all octaves of C are higher powers of the prime number 2) although Chinese intention seems more likely to have been to embody the whole set of five numbers. And if the intention was to incorporate a 'Just' E at 5:4 above C = 64, it would vibrate at 80, as it very nearly does. (Nearly all the E's in the Zeng set favor the smaller 'Just' third.)

Is it conceivable that not only did Chinese craftsmen learn to cast a tuned carillon twenty centuries ahead of us, but that Chinese acoustical physicists anticipated ours by an even greater margin in making absolute frequency determinations? This question invites further exploration.

The Zeng bells as an acoustical tonometer

Robinson and Needham have extensively documented Chinese interest in vibration phenomena, in sympathetic resonance in strings and bells, and in designing bells to produce not only desired pitches but also exquisitely specific tone colors. They quote from the *Mo Tzu*, believed to have been written about 370 BC, shortly after the Zeng bells were cast, on the use of resonators; large jars, sealed with a membrane of fresh skin, were sunk into the ground to 'detect the presence and direction of tunnelling and mining by the enemy besieging a city'. The Chinese, in short, are known to have had the appropriate kind of curiosity to study vibration rates. But how could they have actually

Bell	Pitch	Frequency	Beats/ratio	Summation checks		
53	C2 +23	64.86	] 7.81/1.1204	] 5.13/1.076] 4.06/1.0534] 7.15/1.090	] 15.22/1.2347] 25.80/1.3550	] 33.61/1.5182
54	D2 +20	72.67				
65	D2 +98	76.02	] 3.35/1.0461			
53	E $\flat$ 2 +38	77.80	] 1.78/1.0234			
64	E2 -12	80.08	] 2.28/1.0293			
54	F2 +13	86.07	] 5.99/1.0748	] 4.06/1.0534	] 7.15/1.090	] 15.22/1.2347
65	F2 +38	87.33	] 1.16/1.0135			
63	G2 +46	98.47	] 11.14/1.1276	] 5.38/1.0546	] 25.80/1.3550	] 33.61/1.5182
62	A2 -72	103.25	] 4.78/1.0485			
64	A $\flat$ +38	103.85	] 0.60/1.0058			
63	B2 +77	115.56	] 11.71/1.1128			

Fig. 7. A 'tonometer' experiment for establishing frequencies. If it is possible to tune the bells accurately to a monochord string, and also possible to count beats between pairs of bells accurately, then from the combined data of ratio and frequency difference it should be possible to determine absolute frequency, say within the musical fifth between C2 and G2. Inaccuracies in the tuning do not affect the experiment if the ratios are determined accurately.

determined such rates without the methods and equipment European physicists devised only in the last few centuries?

How the Zeng chime set might be used to determine the frequencies of its own bells is suggested by an experiment devised by J. H. Scheibler (1777–1837) and described in detail by Alexander J. Ellis (Helmholtz, 1885). Working in the octave between A220 and A440 cps, Scheibler made a set of fifty-two consecutive tuning forks which varied from each other by about four beats per second. By sounding consecutive pairs together and summing their differences, counted very carefully through any interval up to an octave, it was then possible to confirm the frequency difference between two boundary tones and thus, from their known monochord ratios, to deduce the actual vibration rates. The fact that the Zeng bells have very clear fundamentals suggests that perhaps they also may be used for just such a 'tonometer' experiment, but two octaves lower where the numbers are much smaller, hence with far fewer instruments.

For the experiment to be successful it must be possible, first, to determine ratios accurately from a monochord tuning; second, for the beats between pairs of bells to be slow enough to count (Ellis found three or four per second most convenient); and third, for the tones to ring for some seconds, to ensure a high degree of accuracy in the counting. For instance, if a low C64 and D72 are struck together and the beat rate of 8 per second is observed, and if their ratio of 9:8 is known from a monochord tuning, frequencies of 72:64 can be deduced immediately from this single experiment. Such laboratory perfection is unreasonable, but Fig. 7 shows how eleven of the lowest Zeng bells might be used in such an experiment, with multiple cross checks to increase accuracy. Whether in fact the acoustical characteristics of the bells do permit this experiment to be done successfully needs to be proven. Huang Xiangpeng writes that he has requested Hubei province to attempt the experiment.

The frequency experiment outlined here requires time to have been measured then as it is now. Needham reports that China adopted an equal twenty-four day about the

beginning of the fifth century BC, just before these bells were cast. The Chinese year-count of 365 1/4 days had already been accurate for a thousand years. How water clocks were calibrated we do not know beyond quarter-hour divisions, too crude for science. But cycles of sixty and three-hundred sixty have always dominated Chinese calendrical and tonal calculations. Under these circumstances, the frequency data cited above can be read as circumstantial evidence for the theory that hours and minutes possibly did enjoy the Old Babylonian subdivisions by 60 which we still use today.

Part 4: Historical and cultural implications

Historical suppression of the Zeng system

Huang Xiangpeng offers an interesting theory as to why the memory of Zeng and its musical system may have been deliberately suppressed a few centuries later when the Confucian ritual was officially adopted by the emperors of a unified China. Zeng was a vassal of the state of Chu, and in the eyes of the mainstream Zhou and Qin cultures, Chu was considered 'a land of the southern barbarians', its music too closely associated with witchcraft. Huang notes that decorations in the tomb of the Marquis of Zeng follow the Chu tradition of depicting 'either ghosts or gods or bewildering animals as performers'. Robinson and Needham have given extensive documentation to the shamanistic aspects of Chinese musical lore. Could the possibility that Confucian traditionalists succeeded in imposing their preferences account for the absence of any historical record of the Zeng bells with their double tones, their five octave span, their accurate tuning, their relative pitch names, their diatonic major scale in the key of C, and of their absolute pitch on an A-flat ground tone? Or is it possible that the burning of the books by decree of the Qin emperor in 213 BC was a greater calamity for historians than some modern scholars have supposed?

The hypothesis being offered here develops Huang's idea in a direction more favorable to actual Chinese history. What needs to be explained, as he clearly perceives, is not so much how the Chinese could have reached so remarkable a level of achievement—they had a thousand years to experiment with bells—but rather why, once possessed, it was forgotten within three hundred years when the first dynastic histories came to be written. In what follows I am suggesting that, far from being an indigenous Chinese development, the tone system on the Zeng chimes belongs to a pan-Eurasian development, probably derived from a far more ancient Mesopotamian past, and that if it was suppressed deliberately, as seems possible, it was done so less out of cultural snobbishness than for the very good reason that it was made obsolete by a stronger, simpler, more logical, and more musically useful Chinese synthesis which then descended through history as 'Confucian' tradition.

Pan Eurasian harmonic structuralism

All of the tonal-calendrical associations in China and Greece can be traced to one 'Just' harmonic matrix implicit in the ancient Babylonian multiplication table and alluded to in the mythology of the *Enuma Elish*, the Babylonian creation myth, at least fourteen centuries before the Zeng bells were cast. Only when the musical octave ratio 2:1 is expanded to 720:360—that is, to a *cycle* of 360 units—are there sufficient integers at the appropriate ratios to define a twelve-tone scale in a way which satisfied the ancient affection for perfect symmetry, a dominant notion in art, science, and political theory. A year of three hundred sixty days is so unlikely a concept for any civilization dependent

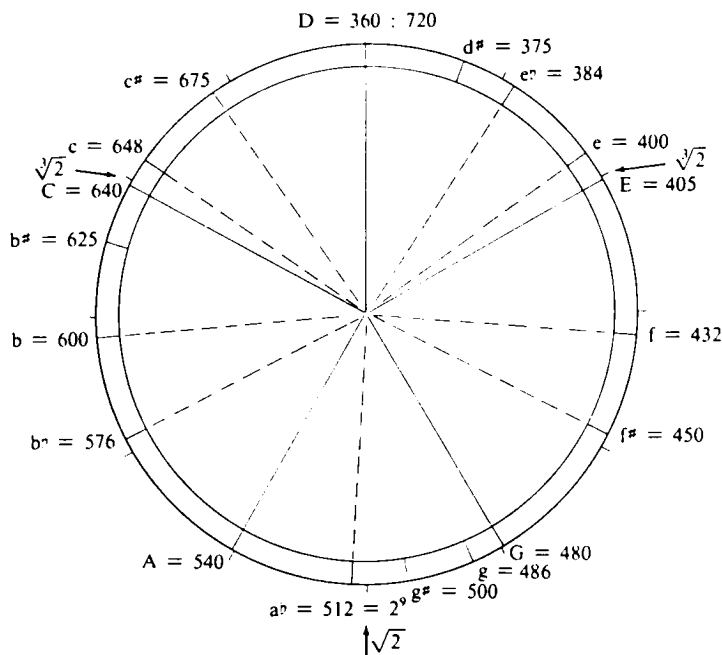


Fig. 8. Plato's tonal calendar. The smallest numerical double with numbers at appropriate ratios for the symmetric major and minor diatonic scales is 720 : 360. This double also contains alternate tunings for C c and E e (small case letters belong to 'Just' thirds). The problematic twelfth zone, ab/g#, lies just off the lower center of symmetry, but as $2^9 = 512$, it is actually the reference pitch from which all of the others are generated. Thus the Chinese pentatonic scale, C G D A E, normally considered to be evidence of a 'spiral of fifths' procedure, is also indigenous to the 'Just' matrix. (From The Pythagorean Plato).

on agriculture that I have no hesitation in suggesting that its widespread use in the ancient world is due solely to harmonical considerations in which the musical octave, 2 : 1, played a normative role. This 720 : 360 numerical 'double' was still useful to Kepler in the sixteenth century AD for the same reason it was needed by Plato and the ancients: it allows the tones of reciprocal major and minor scales to be simultaneously collected into one chromatic octave frame. Within this framing 'double' the five Chinese pentatonic tones have a 'privileged' status. It is immaterial whether we start with the separate scales and bring them together in the matrix, or start with the completed construction and show how the scales are implicit within it; here we shall begin with the overview. This 'calendrical octave' is shown first in the 'tone-circle' of Fig. 8 in the *cyclic* form which dominated all ancient notions of time, and of everything associated with time, and which in Plato's *Laws* serves as both the map of his idealized circular city of Magnesia, and its calendar (McClain, 1978).

The numbers in Fig. 8 are ratio numbers; such a display is thus a logarithmic one, and probably very ancient. The ancient genesis of the numbers can be read in 'the Throne of Marduk' matrix for the multiplication table for 3×5 up to the limit of 720, shown in Fig. 9, where odd numbers defining different pitches are doubled as many times as necessary to bring all tones together within one 'octave double' 720 : 360. (Remember that *even* numbers are merely octave replications.) Here we see the 'throne dais—a seat with a backsupport' which made Marduk's countenance 'shine exceedingly' (McClain 1976; 1982). This throne required two years (i.e. $2 \times 360 = 720$ days) to build, the 'sar'

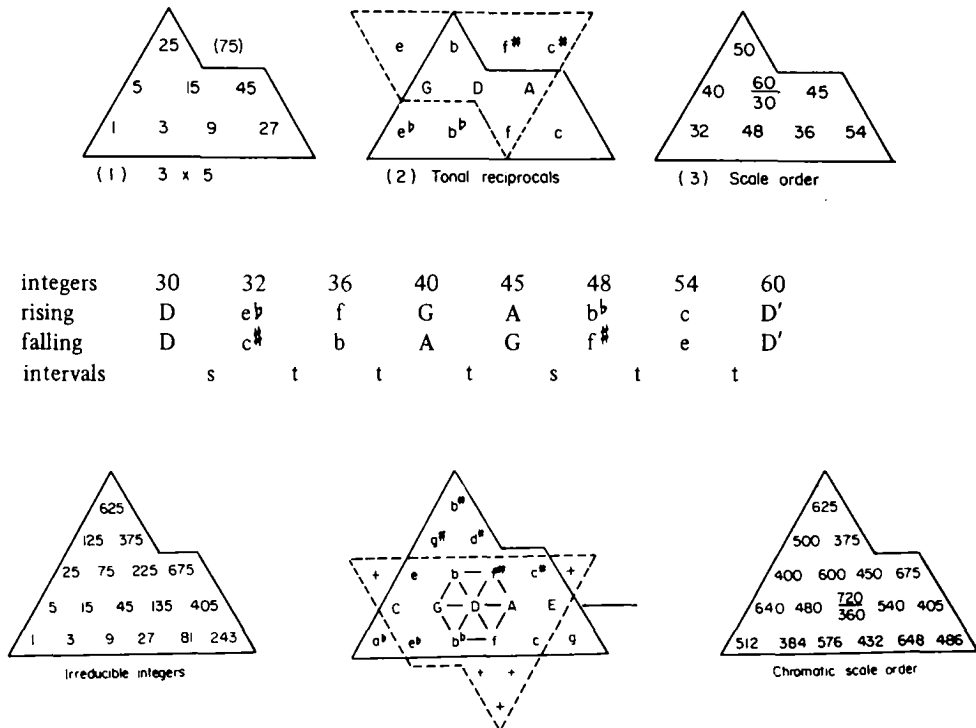


Fig. 9. Platonic-Babylonian-Zeng matrix. The 'Just' tuning matrix for the 720 : 360 octave begins with the multiplication table for 3×5 , gaining one new musical fifth with each power of 3, and one new musical third with each power of 5. These odd, 'male' numbers are then doubled as many times as necessary (by the principle of 'octave equivalence') to bring them within one numerical double. Rotation of the table by 180° discloses which tones possess reciprocals within the boundary. Notice how the Chinese pentatonic scale, normally thought of as a spiral of fifths, actually comes to birth along the central axis of this Platonic-Babylonian 'Just' matrix.

of 720 being the unit with which bricks were counted. The numbers have double arithmetical meanings as both multiples and submultiples (of any basic unit whatever). They have double tonal meanings in rising and falling scales. They also have the double acoustical implications of wave-length and frequency—but only after these more sophisticated notions arise in physics.

Notice that only four tones surrounding the center of symmetry—here labelled D because it is the centre of symmetry in our modern tone systems and on our modern keyboards—are linked to it directly in the spiral of fifths. The pentatonic scale material—C G D A E—is the very heart of this matrix. Only this central line of tones remains invariant under arithmetical reciprocation (tested by rotating the matrix 180° degrees). Without knowing it, Li Chunyi has used the alignments of this ancient Platonic-Babylonian graph (Fig. 3, above) to show the pattern of musical thirds and musical fifths implied by the relative pitch tone-names. The Zeng diatonic scale—rising C D E F G A B C—is the tonal-arithmetical 'major' reciprocal of Plato's 'true Hellenic mode', the 'minor' Greek Dorian, falling E D C B A G F E, inseparably bound together in perfect inverse symmetry. When these tonal reciprocals (Plato's 'twins') are transposed to the center of symmetry on the tone D, their sequence of wholetones and semitones can be defined by smaller tone-numbers in the octave 'month' cycle 60 : 30.

D	c	bb	A	G	f	eb	D	Greek Dorian
D	e	f#	G	A	b	c#	D	Zeng diatonic
60	54	48	45	40	36	32	30	ratios

The twelfth tone, missing from this 'Just' set, is the *Huang-chung* A-flat from which it is generated as 1×2^n , in this case 2 to the 9th power = 512 (notice its location and transformations in the lower left corner of the matrices in Fig. 9). When projected into the larger 720:360 double, two alternate tunings, C and E, provide the traditional Chinese pentatonic tunings for the spiral of fifths. (Small case letters here denote small 'Just' thirds of 5:4.)

Our Platonic-Babylonian matrix is synthesizing all of the Chinese tonal materials, even to providing an excuse for the naming by major thirds, for emphasizing sequences of major and minor thirds, for conceiving *shang* D ('minister') of exceptional importances (it is the center of symmetry, the guiding principle), for grounding the pentatonic scale on C, and for grounding *Huang-chung*, tonic of the absolute pitch system, on A-flat, a major third lower. To anyone familiar with Plato's Pythagorean constructions, the Zeng chimes are a perfect embodiment of its basic materials. It may be doubted whether any mutual aspect should be considered 'merely' coincidental. What is really embodied is an ancient and universal tonal cosmology, not an idiosyncratic ethnic one.

That this 'Just' tuning system with its calendrical correlations was once well-known in China despite its suppression in the official histories might have been inferred earlier from many other clues. The prevalence of 60 and 360 in later Chinese acoustical theory, despite the *irrelevance* of these numbers to the official 'Confucian' spiral of fifths, is a reminder; so is their persistence in Chinese calendrical calculations despite a far more accurate Chinese year-count. The presence of the 'Just' tone-ratios of 5/6, 4/5 and 3/5 along with those of 3/4, 2/3, 1/2 in the oldest Chinese metallurgical text, and one giving the proportions of copper in casting bronze bells (thought by Robinson and Needham to date no later than the third century BC), is an even stronger clue that such ratios were probably present also among the bell tones. But far earlier by a thousand years, the most ancient and venerated of Chinese stringed instruments, the *ch'in*, a long zither which evolved no later than the fourteenth or fifteenth centuries BC and for which there is oracle bone evidence as early as the eighteenth, has a series of thirteen ivory or mother-of-pearl discs inlaid symmetrically along one side of the soundboard at appropriate 'Just' ratios for the harmonics. (Their distances are 1/2, 1/3 and 2/3, 1/4 and 3/4, 1/5, 2/5, 3/5, 4/5, 1/6 and 5/6, and 1/8 and 7/8.) The string divisions by fifths and sixths contain pure major thirds never present in the approved spiral of fifths. Sets of lithophones with carefully tuned 'Just' and 'spiral of fifths' musical thirds are reported to date from as early as 900 BC (Kuttner, 1965). In short, the suppression of the 'Just' ratios was never complete; modern musicology, under the influence of the dominant tradition, has simply been trying to ignore them.

The great charm of numerical correspondence between scale, calendar, and the lives of men (avatars and 'guardians' start their ministries at 'age 30') in cycles of 30, 60, and 360 could have had limited meaning for a fifth-century Chinese science which had already known for a thousand years that the year had 365 1/4 days, or to a Greek science now well on the way to understanding the calendrical truth. And in the essentially melodic genre of both Chinese and Greek music, the niceties of pure thirds of 4:5 versus 'ditone' thirds of 81:64 are also essentially irrelevant tonally. The spiral of fifths which became standard in China as in Europe is musically far simpler, especially under the influence of a vigorous system of transposition, because fewer strings must be retuned

(or bells and pipes replaced) when changing from one key to another. Perhaps equally important, spiral fifths are closer to a true cyclic tuning (i.e. to twelve-tone equal temperament). 'Just' tuning, with its beguiling numerical correspondences, had become a logical monstrosity and a tonal handicap; it deserved suppression. What was transmitted through history instead was a 'Confucian' simplification stripped of every tonal and arithmetical excess until, in time, it also acquired much useless baggage from excessive veneration. The laconic old Chinese rule for the spiral of fifths—'add or subtract one-third'—has an unsurpassable simplicity which long ago should have been recognized as the elegance of mature mastery, not primitiveness. And if the scale is essentially pentatonic, five—not twelve—relative pitch names are quite sufficient. The lost names sustained a system no longer relevant.

From the perspective established here—that the Zeng chimes embody an ancient cosmological vision—it is now possible to show how they may have led to an even more interesting Chinese development, one probably open only to people who could make large bells like those of Zeng.

The new Chinese musical synthesis

The correlation of calendar and scale, standard pitchpipes, bells, weights and measures of the empire and the lives of men was an harmonical passion in China. As early as the first century BC it was affirmed that,

the pitchpipes and the calendar give each other a mutual order, so closely that one could not insert a hair between them (Needham, 1956).

The theory proposed here is that the same technology which created the Zeng chimes—tuned intentionally to 'philosophical pitch'—made possible a coordination of all other Chinese measures with the actual Chinese year-count of 365 1/4 days.

Once it is possible to tune the pentatonic scale to known frequencies, it is a simple matter to set the monochord moveable bridge to F-sharp above middle C vibrating at 360 cps.; it is the major third at the vibration ratio 5:4 above *shang* (re) D, the D octaves being 72, 144 and 288, or the string length ratio 4:5. And once the tone F-sharp/G-flat is set at 360 cps, the new and more accurate 'calendrical' one can be determined at 365 1/4 (or thereabouts) directly from their beat rate of about 5 per second. The monochord also makes alternative procedures available, and those thirteen hundred *se* bridges found in the tomb remind us of the wealth of experience Zeng musicians had with string division.

In the Zeng chimes, the *shang-jue* F-sharp, bell number 40 in set IIb, actually vibrates 365.4 cps. From such a bell it is possible to tune the standard *Huang-chung* pitchpipe simply by cutting it to a length which allows it to match the bell. (The pipe pitch is very sensitive to temperature, so we should not be dogmatic about precise pipe frequencies.) From the length of this fundamental pipe, always made very narrow, thus difficult to blow but therefore relatively stable in pitch, the Chinese foot was traditionally derived, the pipe being conceived as nine-tenths of the foot. From the volume of grain the pipe could hold, the standard weights and measures of the empire were derived. Such procedures by the bureau of standards promised that the empire was in harmony with the world—until some national calamity proved otherwise and a complete revision had to be undertaken. Dozens of such changes are chronicled, centered around the F and F-sharp above middle C.

But granting now the possibility that Zeng acousticians could consciously tune a pipe

to the 'cosmological F-sharp' vibrating 360–365 cps, where is the evidence that this was ever adopted as the national standard? Let us look first at the bells, then at the pitchpipes.

No special engravings are reported on bell number 40, which vibrates 365.4 cps to acknowledge its cosmological status, but the importance of this pitch is emphasized in other ways. *This is the only Zeng key besides C to be given a complete set of its own bells.* The nineteen bells in the top row of the chime set celebrate this calendar key. The thirteen bells in sets Ib and Ic (once unified, and fourteen in number) have this particular F-sharp/G-flat as their ground tone (see bell number 19), where as *gong* (do) it now vibrates at 362.2 cps, and it is on these bells that the multiple cross-references to the tone names in the key of A-flat are engraved (see Table 3). Now it is reported that Zeng tones were correlated, via the inscriptions, with the states of Chu, Jin, Zhou, Qi, Shen, etc., and that, 'in addition to the information directly obtainable, many materials were hidden in a very tricky manner' (Lee, 1980). Is it possible that the calendrical correlation was made quietly by tonal embodiment, by the labelling, and by the physical arrangement of the bells, but with implications evident only to the craftsmen who guarded this art? Is it 'merely' a coincidence, I wonder, that the higher fundamental on the *ho* type 'gift' bell sounds the proper 'Just' *Huang-chung* A-flat while the lower fundamental sounds F-sharp, two octaves below the proper calendrical standard?

The more interesting evidence that China actually adopted the cosmological F-sharp standard is hidden in the oldest pitchpipe formulas, waiting to be tested. The ancient Shang and Hsia foot (third millenium BC), traditionally divided into tenths, was 25.525 cm; nine-tenths of this length, the traditional pipe formula, gives a pipe length of 22.97 cm. The only ancient diameter formulas are 1/27 and 1/30 of the length (depending on interpretation of the texts). These diameter differences are insignificant; both produce a pipe so narrow that it is difficult to blow, but consequently difficult also to vary in pitch by adopting different manners of blowing. The oldest pitchpipe calculations to survive are those of Ssuma Ch'ien (c. 90 BC), in the Han dynasty when the standard foot was much longer, being 28.361 cm. But Ssuma Ch'ien interpreted the ratio number 81 as 81 tenths of an inch, and 0.81×28.361 is also 22.97, so that his *Huang-chung* pitchpipe had the same length as the legendary ancient one. Now musicologists tend to assume that such a pipe sounded the F above middle C, but if the length of this ancient pipe was measured *externally*, as would have been appropriate for a visible symbol of 'natural law', but the pipe was closed at the lower end by a natural node in the bamboo, as Chuang Pen-li (1965) suggests that the earliest pipes were, its effective *inner* length would have been reduced by two or so mm. This 'nodal' end correction raises the pitch of the pipe. On such a pipe and at a moderate temperature, as anyone can test for himself, the 'year-count' can be blown easily. We have already seen that our pitch measuring standard should be *lowered* by thirty-eight cents to adjust for 'philosophical pitch'. Thus the correct interpretation of the ancient pitchpipe formula for *Huang-chung* should probably be F-sharp/G-flat at 360–365 cps.

My pitchpipe argument, which invites only a trivial modification of current Chinese thinking, must be developed in detail elsewhere. The pipes have been studied intensively for the last century, and with immense frustration, but never to my knowledge from this slightly different perspective. No other way of measuring pipe lengths can produce a scale even remotely in tune. Measured in the naive way I suggest, an element of serendipity makes possible an impressively well-tuned twelve-tone wind instrument. Within two generations of Ssuma Ch'ien, Ching Fang, the greatest of early Chinese acousticians, was sophisticated enough to declare the pitchpipes unreliable, and to insist on string measurements in acoustical theory (McClain, 1979).

Ssuma Ch'ien's pipe formula supports the theory that the Zeng bells, or a comparable alternative, did make possible the correlation of the pitchpipes with the year-count. This new tonal standard might easily have set into motion the musical mythologizing of early Chinese history which has remained a baffling barrier to later scholars trying to unravel fact from fiction. The old *Huang-chung* of the bells was A-flat, as it should have been to embody the ancient cosmology; the new *Huang-chung* of the pitchpipes is F-sharp/G-flat, as it should be for a perfect correlation with the calendar. Thus we have discovered a good reason for burning the old music books and mythologizing Chinese history around the story, set in the twenty-eighth century BC, that China's ancient standards came from a set of pipes 'brought home from the western mountains'. The myth, encapsuled history, preserves fundamental truth.

Conclusions

1. The Zeng chimes, their method of display, their naming system, and their tunings evidence a 'structural' maturity, tonal complexity, and technological virtuosity beyond anything we have hitherto suspected in the music of the ancient world. They command profound respect on a philosophical-musicological level as well as on a technological one. Contemporary fifth century classical Greece, which we are in the habit of venerating, left no artifacts of comparable musical value.
2. The tonal cosmology of ancient China now appears to have been based on a 'Just' tuning system in the theoretical octave 720 : 360, used intensively less than a century later by Plato in his *Republic*, *Critias*, and *Laws* dialogues, and possibly inherited from ancient Babylon. It was later suppressed in favor of a simpler Chinese 'Confucian' synthesis around the spiral of fifths.
3. This new Chinese synthesis probably made the F# above middle C, vibrating at the year-count of 360–365 cps, its *Huang-chung* fundamental, an accuracy owed to the bells. History was then mythologized around pitchpipes with the desired accuracy, and conflicting musical ideas were suppressed.
4. The theory of a common human bias in favor of tonal symmetry gains new support from the pattern of interlocked fourths and fifths in the Babylonian formula 6 : 8 :: 9 : 12 (C–F–G–C and its transpositions), the foundation not only for Greek musical theory but also, we now know, for the Zeng chime scales.
5. Confirmation that the prevailing diatonic pattern in China as well as India, Greece, and Babylon is that of the C major scale or its inverse (Greek Dorian) raises new questions about the continuity of psycho-physical tonal norms across barriers of time, space, and style. The ratios and symmetries of music very likely 'reflect the order and balance in a strange mirror that is our own brain' (Fischer, 1984).
6. Since the ancient mythical imagination contained a strong component of tonal-arithmetical allegory, based on a substantial mastery of elementary acoustical physics, ancient cultures therefore need to be studied by an anthropology educated in the harmonical sciences of the ancient world.
7. The excessive bias of Western scholarship in favor of philological methods, and to the neglect of organological evidence, has been a crippling influence on sinological studies, protested before (Kuttner, 1965). Had the Zeng chimes become known to Herodotus, their Greek contemporary, we might have heard about them long ago as the eighth wonder of the ancient world. The national pride and joy which radiates now through Chinese reports about the bells is entirely appropriate. The Chinese know—as the Western world cannot know until the research material is made fully available in our languages, with adequate recordings of the chime tones—what an incredible level of

musical competence their ancestors reached in the fifth century BC, and how incommensurate that achievement is with what the world has been taught to believe.

Acknowledgements

The author wishes to acknowledge considerable personal debts to Barry S. Brook, Ming Shui Hung, Siegmund Levarie, and Richard Sacksteder, his colleagues in the City University of New York, and to Chi-Jen Chang, Fritz Kuttner, Robert Stierer, Theodore Rossing, Jan-Nin Yu, and Huang Xiang-peng. *Chinese Music* has graciously consented to the publication of Figs 1 and 2.

References

Yinyue Yanjiu, 1981 (in Chinese)

- Huang Xiangpeng. A preliminary study of the musical system in the inscriptions of the Bell-Chime and Stone-Chime.
 Li Chunyi. A textual research of the inscriptions on the Bell-Chime.
 Tan Weisi and Feng Guangsheng. Approaches to the musical properties of the small bells in the uppermost row of the Bell-Chime.
 Wang Xiang. An exploration of the temperament of the Bell-Chime excavated from Zeng Houyi's tomb.

Chinese Music (in English)

- Huang Xiangpeng. The mystery of dual-pitch system in Chinese *bian-zhong*. Vol. 3, no. 3, September 1980.
 Huang Xiangpeng. *Chu-shang* elucidated. Vol. 3, no. 1, March 1980, and no. 3, September 1980.
 Lee Yuan-yuan. The music of the *Zenghou zhong*. Vol. 3, no. 1, March 1980.
 Lee Yuan-yuan. Music theory suggested by *zhenghou zhong* and its inscriptions. Vol. 3, no. 3, September 1980.
 Ma Cheng-yuan. Ancient Chinese two-pitch bronze bells. Vol. 3, No. 4 December 1980, and vol. 4, no. 1, March 1981.
 Anon. Acoustic Design of the *Bian-Zhong*. Vol. 4, No. 2, June 1981.
 Tong Shibü. The *Se* bridges of *Zenghouyi*. Vol. 6, no. 3.

Other sources

- Chang, Kwang-Chih (1980). *Shang Civilization*. New Haven: Yale University Press.
China Reconstructs (1979) 28 (5 May).
 Chuang Pen-Li (1968). *Panpipes of Ancient China*. Nanking: Institute of Ethnology Academia Sinica Monographs No. 4.
 De Nicolás, A. (1976). *Meditations Through the Rg Veda*. Stony Brook: Nicolas Hays Ltd.
 De Nicolás, A. (1982). *J. soc. biol. Struct.* 5(3).
 Fischer, J. (1984). *J. soc. biol. Struct.* 7(4).
 Helmholtz, H. (1885/1954). *On the Sensations of Tone*. New York: Dover.
 Hua Jueming & Jia Yunfu (1982). *Historia Scientiarum*, no. 23, The History of Science Society of Japan.
 Kilmer, A. D., Crocker, R. & Brown, R. (1976). *Sounds from Silence* (with recordings). Berkeley: Bit Enki Publications.
 Kuttner, F. (1965). *Ethnomusicology*, 9(1).
 Kuttner, F. (1964). *Ethnomusicology* 8(2).
 Larson, G. J. (1982). *J. soc. biol. Struct.* 6(2).
 McClain, E. (1976). *The Myth of Invariance*. New York: Nicolas Hays.
 McClain, E. (1978). *The Pythagorean Plato*. New York: Nicolas Hays.
 McClain, E. (1979). *Ethnomusicology*, xxiii(2), May.
 McClain, E. (1982). *J. soc. biol. Struct.* 5(3).

-
- McClain, E., with Ming Shui Hung (1985, in progress). Kao Yu's Commentary on Tone Names. Nakaseko, (1957). *J. music Theory*, 1(2, November).
- Needham, J. (1954–1962). *Science and Civilization in China*. Cambridge: Cambridge University Press. Vol. 1, 1954; Vol. 2, 1956; Vol. 3, 1959; Vol. 4, part I (with Kenneth Robinson) 1962.
- Quian Hao, Chen Heyi & Ru Suichi (1981). *Out of China's Earth*. New York: Abrams, and Beijing: China Pictorial. September 1983.
- Rossing, T. D. (1984). *American Scientist* 72(5).
- Scientific American* (1984) 251 (December).
- Stieglitz, R. R. (1982). *J. soc. biol. Struct.*, 5(3).